A Unified Framework for Defining and Measuring Flexibility in Power System

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***Abstract*—Flexibility is a widely used term in planning process and real-time operation. The existing research on flexibility uses different techniques to study flexibility property from different aspects. While studying a property from various viewpoints increases the understanding on the subject, we need a consistent theoretical framework to consolidate the ideas generated in the field, compare and contrast results, and build on for future analysis. Based on the insights of the nature of flexibility, this paper proposes a unified framework for defining and measuring flexibility in power system. Under the proposed framework, we propose a flexibility metric which evaluates the largest variation range of uncertainty that the system can accommodate. Such a metric takes into account transmission network and system operations constraints, which are critical to assessing flexibility, but are often ignored in literature. A robust optimization technique is used to calculate the proposed metrics. While the illustrative example presented in this paper focuses on the flexibility in real-time system operation in the presence of wind and load uncertainty, this framework can generally be applicable to long-term studies such as system planning.**

***Index Terms*—Flexibility, flexibility metrics, robust optimization.**

I. NOMENCLATURE

Parameters

c_j Production cost of a corrective control unit (CCU) j (\$/MW).

$\bar{C}_T$ Cost threshold associated with the response time T (\$).

$F_l^{\max}$ Capacity limit of transmission line l (MW).

$p_{j,0}$ Power output of a CCU j at the initial time 0 (MW).

$p_{j,T}^{\min}, p_{j,T}^{\max}$ EcoMin and EcoMax of generator j at time T (MW).

$SF_{n,l}$ Shift factor of node n to line l.

$\Delta_j^{\rm dn}, \Delta_j^{\rm up}$ Downward and upward ramp rates of a CCU j (MW/min).

Variables

$d_{n,T}$ Net load at node n at time T (MW).

$d_{n,T}^{\rm LB}, d_{n,T}^{\rm UB}$ Lower and upper bounds of the variation range of net load at node n at time T (MW).

$p_{j,T}$ Power output of a CCU j at time T (MW).

$u^{\rm LB}, u^{\rm UB}$ Lower and upper bounds of the variation range of uncertain parameter u.

Functions and Sets

$j(n)$ Index of resources j which is located at node n.

CCU Set of the CCUs.

TL Set of the transmission lines.

RAND Set of the net load subject to uncertainty.

$\sigma(u^{\rm UB}, u^{\rm LB})$ Function that measures the size of the hypercube $[u^{\rm LB}, u^{\rm UB}]$.

$r(d_{n,T}^{\rm UB}, d_{n,T}^{\rm LB})$ Radius of the Chebyshev ball of the hypercube $[d^{\rm LB}, d^{\rm UB}]$.

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II. INTRODUCTION

As MORE variable resources are integrated into the electric power system, supply and demand uncertainty increases dramatically. This requires the system to have the ability to react to a sudden change and accommodate new status within acceptable time period and cost. Therefore, the notion of flexibility recently has been drawing extensive attention in the power industry. In most cases, system flexibility is obtained via either the provision of increased reserve, the construction of transmission, alternative market design, or operational procedures, as investigated in renewable generation integration studies [1], [2] and [3]. To determine the additional reserve needed to provide flexibility, a Monte Carlo chronological simulation has been proposed in [4]. In industry, some ISOs consider procuring flexibility by introducing ramping products. For example, the Mid-Continental ISO [5] has introduced a market ramping product based on the expected scarcity of ramping resources in the short-term. CAISO also establishes a flexible ramping product [6]. Additionally, Lu *et al.* [7] propose that the flexibility requirements should include reserve, ramp rate, and ramp duration constraints.

While the literature is not lacking in the number of flexibility measures provided for different aspects of flexibility using various approaches, there is a lack of a broader framework that encompasses these different concepts and techniques, enables us to analyze flexibility from a richer perspective, and provides a foundation for future research. The goal of this paper is to provide such a framework based on the common fundamental elements that define the flexibility in different aspects of power systems. A unified flexibility framework for power systems will allow flexibility to be explicitly considered in the design of the system from both short-term and long-term perspectives. It will also make it possible to quantitatively compare across different flexibility options open to decision makers that would otherwise not be possible with a multitude of frameworks based on different assumptions and varying forms. A unified framework can provide the ability to value different aspects of flexibility of a system, and enable the integration of flexibility concept in the system design.

The increasing amount of uncertainty necessitates the introduction of more flexibility in the power systems. Hobbs *et al.*[8] suggest that the importance of flexibility is well recognized, but the concept has yet to be clearly defined and quantified. The previous studies of flexibility of power systems can be divided into two groups based on their target applications. One focuses on long-term planning while the other targets real-time operations. A comprehensive review of the research on both long and short-term flexibility metrics is performed in [9].

In generation planning, Lannoye *et al.* [10] define flexibility as the ability of a system to deploy its resources to respond to changes in the demand not served by variable generation. They suggest the insufficient ramping resource expectation (IRRE) to assess flexibility of a system, similar to the LOLE for capacity adequacy. In [11], flexibility is integrated in long-term generation planning models based on an enhanced unit commitment model that determines an optimal generation portfolio. In transmission design, the authors of [12] and [13] define flexibility as the attribute of the transmission system to keep up a desired standard of reliability, at reasonable operation costs, when the generation scenarios change. They also develop technical and economical flexibility indices of transmission system from the statistical point of view.

From a short-term operational perspective, Menemenlis *et al.* [14] propose a flexibility metric using balancing reserves via simulating various demand and variable generation production scenarios. Bouffard and Ortega-Vazquez [15] see flexibility as the potential capacity to be deployed within a certain timeframe and associate flexibility with reserves. Studarus and Christie [16] propose an operational flexibility metric that reflects the available system operation range relative to the uncertainty range of current and future system state without considering transmission network. It is suggested in [17] that power capability for up and down regulation, energy storage capability, power ramping capability, and power ramping duration should be used as metrics for operational flexibility. Ma *et al.* [18] propose a flexibility index to estimate the technical flexibility of both the individual generators and the generation mix based on generators' ramping capability and generating capacity.

Most of the above flexibility definitions and metrics proposed pertain to particular aspects of power systems. Many of the assumptions underlying some of the metrics make their field of application very narrow. In this paper, we identify four elements, namely, time, uncertainty, action, and cost that are common to the flexibility literature in power systems. These four crucial elements serve as a basis for constructing effective measures of flexibility that can be applied to a wide range of areas. The proposed framework is a systematic approach to developing metrics for flexibility from its foundational elements, rather than to start from a metric that applies to a particular case of interest. As a result, the essence of flexibility studied in the aforementioned papers can be readily captured under the proposed four-element framework.

Flexibility metrics can be probabilistic or deterministic. The probabilistic metric measures the system's ability to meet certain reliability or security criteria in a probabilistic fashion, e.g., the IRRE proposed in [10], and the probability of simulated scenarios being satisfied by an operating strategy in [14]. The probabilistic metric is typically obtained by using simulation based approaches. The deterministic metrics represents the flexibility information deterministically, but still recognizing the stochastic nature of system uncertainty. The deterministic metric can be translated into simple language such as "does the system have enough flexibility in the next hour?" in [16], or "how much flexibility does a system or a resource have?" in [18]. The advantage of deterministic metric over a probabilistic one is that the former is more intuitive than the latter. Consequently, it is easier for decision makers to make decisions based on the deterministic metrics.

In this paper, we construct a flexibility metric that reflects the largest variation range of uncertainty that the system can sustain by using the four-element framework. More specifically, the largest variation range of uncertainty that the system can reliably accommodate is determined first. Then, it is compared with a target variation range of uncertainty that reflects decision makers' risk level to evaluate the flexibility of a system. The proposed flexibility metric is deterministic. Besides being an intuitive measurement, the proposed metric has the following three attractive features that distinguish it from the existing deterministic metrics.

1) It explicitly considers the impact of transmission network and system operation constraints. Lannoye *et al.* [9] underscore that these constraints actually play a significant role in determining the flexibility of a system. Congestion in the transmission system can lead to a decrease in flexibility of a system, so their inclusion is critical to assessing the overall flexibility of a system. However, very few of the existing deterministic metrics handles transmission network and system operation constraints, with the exception of [19]. Reference [19] suggests using sensitivity analysis to measure the flexibility provided by each component in the system. However, the proposed sensitivity analysis can only be applied to deterministic problems. Therefore, such an approach cannot handle power system with uncertainty. In addition, the sensitivity analysis proposed in [19] cannot be applied to integer problems, which are often used to model unit commitment actions in power system. In contrast, the proposed methodology in this paper can deal with corrective actions with integer variables. Transmission network and system operation constraints are also considered in [20], [21]. However, the flexibility metric studied in [20] requires time series models, and is a probabilistic metrics, instead of a deterministic one. Reference [21] focuses on designing a system with optimal flexibility, but it does not provide a flexibility metric.

2) It provides information that can help to improve system flexibility. As demonstrated in the numerical experiments, we can develop zonal ramping requirements by using the proposed metric. A possible flexibility shortage in the future can be avoided by operators proactively assigning ramping requirements at the areas where the shortage may occur.
3) It demands moderate stochastic data information and requires little computational effort. The range that contains all possible realizations of uncertainty is known as the uncertainty set in robust optimization [22], [23], and [24]. We employ the robust optimization solution methodology developed in [25] to obtain the largest variation range of uncertainty or uncertainty set that the system can accommodate. Such a solution methodology is shown to be computationally efficient in the numerical examples that use the large-scale ISO New England power system. Since uncertainty is characterized by the variation range, we do not need to process probabilistic information of uncertainty, which is usually hard to obtain.

The main contributions of this paper are summarized below.
1) We introduce a unified flexibility framework that provides a systematic way to define and quantify power system flexibility in the presence of uncertainties. The flexibility definition is based on four essential elements that are common to various applications of flexibility.
2) Using the proposed framework, we develop an intuitive deterministic flexibility metric that explicitly incorporates transmission and operational constrains of the system. This metric can also provide useful information to improve system flexibility. It can be calculated using a robust optimization technique instead of running extensive simulation.
3) As an example, the proposed flexibility metrics is applied to real-time operation in the power system operated by ISO New England.

This paper is organized as follows. Section III defines the term flexibility. Section IV describes the flexibility metric. Section V briefly introduces the solution methodology. Section VI presents a real-time application of the proposed flexibility metric. Section VII is the conclusion.

III. Definition of Flexibility

Flexibility at a given state is the ability of a system to respond to a range of uncertain future states by taking an alternative course of action within acceptable cost threshold and time window. Flexibility is an inherent property of a system. The following four elements are identified as the determinants of flexibility:
1) time (T);
2) actions (A);
3) uncertainty (U);
4) cost (C).

The first three elements are affected by power system operational criteria while the last element is determined by economic criteria. Next, we will describe each element in details.

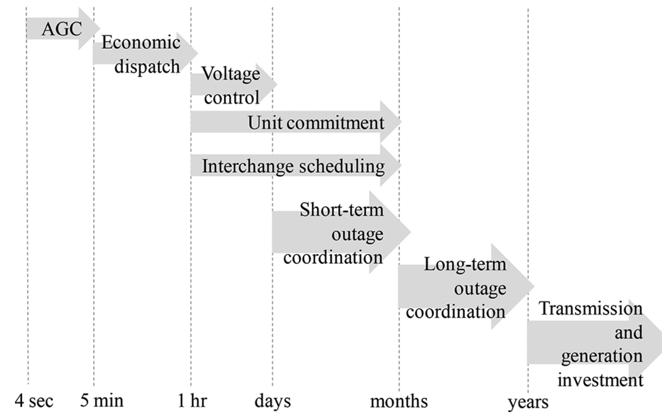

Fig. 1. Available control actions associated with different response time windows.

A. Time (T)

The response time window T indicates how fast the system is expected to react to state deviations and restore the system to its normal state. The time window can be seconds, minutes, hours, days, and months depending on the purpose of study. Based on the selected response time, a system may have different flexibility levels. Shorter time windows focus on the short-term operational flexibility, which indicates a system's timely response to emergency in minutes or hours. Longer time windows focus on long-term planning, which shows a system's ability to cope with changes such as generation mix, regulatory policy, and electricity consumption pattern changes, in years. A power system may show more flexibility in longer time periods, while lacking it in shorter time frames. For example, a system might have sufficient capacity to cope with demand growth in a period of a year, but not enough to adapt to daily load fluctuations. Therefore, the time horizon has to be determined when we compare and evaluate system flexibility.

B. Action (A)

The set of corrective actions A represents the corrective actions that can be taken within the response time window under certain operating procedure. Therefore, the corrective actions set depends on the response time window, i.e., $A(T)$. Fig. 1 illustrates the typical corrective actions associated with different response time windows. For instance, if $T = 1$ h, the corrective action set may include such actions as voltage control, commitment of units, and interchange scheduling. The size of the available corrective action set reflects the diversity of corrective actions. The larger the set $A(T)$ is, the more options operators have to respond to unexpected events. In turn, the response cost can be reduced or more uncertainty can be accommodated. Operating procedure changes or technology improvement will affect the corrective action set.

C. Uncertainty (U)

Uncertainty is the lack of complete information of the state of the system in the future. There has always been uncertainty in power systems operations and planning. Uncertainty is traditionally associated with the likelihood of failure of components, forecast errors, or strategic gaming behavior of market participants. In recent years, the increase in variable generation creates new sources of uncertainty in the system because its output cannot be perfectly foreseen. In addition, with the development of smart grid, the unexpected emergent behavior [26] may also arise in the complex power system. The magnitude of uncertainty determines how much flexibility a system requires to handle uncertainty and how flexible a system is. For example, the uncertainty considered under the $\mathrm{N}-1$ criterion U_{n-1} is the loss of any single transmission or generation elements whereas the uncertainty considered under the $\mathrm{N}-2$ criterion U_{n-2} is any combinations of two random outages of transmission or generation elements. A system that is flexible with respect to U_{n-1} may not be flexible if U_{n-2} is considered. We call the variation range of uncertainty that the system aims to accommodate the target range. The target range reflects decision makers' risk preference, and is subjectively set by operation or planning criteria. The larger the target range, the more conservatively the decision makers design or operate the system.

D. Cost (C)

The response cost C depends on the corrective action $a(\in A)$. This implies that the cost is a function of a, i.e., $C(a)$. In some cases, there can be a response cost threshold $\bar{C}$, which sets an upper bound on the cost to cope with the uncertainty realization. In other words, $C(a) \leq \bar{C}$. As a result, the cost threshold puts restriction on the available corrective actions in addition to the physical limitation associated with the time scales as illustrated in Fig. 1. If the cost threshold is infinitely large, then there is no restriction on corrective actions associated with the cost limitation. If the cost threshold is low, some corrective actions become uneconomical and will not be taken into consideration. In some other cases, the objective of a decision maker can be minimizing the response cost, i.e., $\min_{a\in A} C(a)$. Under this objective, the most economic corrective actions are sought in response to uncertainty.

With the four essential elements presented above, we use [11] as an illustrative example to show that the flexibility problem studied in the context of long-term generation planning can be addressed by using the proposed four-element framework. In particular, in [11], (T) the response time window is set to be one year to reflect the variations of uncertainty that occur naturally over the course of a year; (A) the corrective actions include whether building a unit or not, unit commitment and economic dispatch; (U) the uncertainty taken into account is the load level, load profile and renewable generation; (C) the authors consider the sum of dispatch, commitment and investment costs, and try to minimize it. By running an enhanced unit commitment model, the most economic generation portfolio can be obtained to meet the flexibility needs in various load and wind generation scenarios in [11]. By properly defining the four critical elements, we can easily apply the proposed framework to other applications in the flexibility literature.

IV. Flexibility Metrics

Here, we use the elements identified in the previous section as basis to construct a flexibility metric. In particular, we first identify the largest variation range of uncertainty within which the system can remain feasible under given response time horizon and cost threshold. The flexibility metric is obtained by comparing the largest variation range with the target range to reflect excessive availability of the system relative to the target variation range.

The first step to measure flexibility is to clarify the response time window, cost threshold, and the target variation range. The first two elements indicate the time horizon of interest, and the economic boundary, respectively. The available corrective actions naturally follow once these two elements are chosen. The target range serves as a basis for evaluating flexibility, reflecting decision makers' risk level.

Given a response time window T, we assume that the target variation range $\bar{U}_T$ that decision makers wish to accommodate at the time T. The target range $\bar{U}_T$ can be characterized by a hypercube as follows:

$$\bar{U}_T = \{u|\bar{u}^{\mathrm{LB}} \leq u \leq \bar{u}^{\mathrm{UB}}\}$$

where u is an n-dimensional vector, representing n uncertainty sources in the system. The parameters $\bar{u}^{\mathrm{LB}}$ and $\bar{u}^{\mathrm{UB}}$ represent the lower and upper bounds. To derive the flexibility metric, we want to obtain the largest variation range of uncertainty that the system can accommodate. The largest variation range problem can be formulated in an abstract form as follows: for a given response time window T, and a response cost threshold $\bar{C}$ we have

$$\max_{u^{\mathrm{LB}},u^{\mathrm{UB}},a(\cdot)} \sigma(u^{\mathrm{UB}}, u^{\mathrm{LB}}) \tag{1}$$

$$\text{s.t.} \quad Aa(u) + Bu \leq b, \quad \forall u \in [u^{\mathrm{LB}}, u^{\mathrm{UB}}] \tag{2}$$

$$c^T a(u) \leq \bar{C}, \quad \forall u \in [u^{\mathrm{LB}}, u^{\mathrm{UB}}]. \tag{3}$$

The objective function (1) of the above problem is to maximize the size of the variation range of uncertainty. The measurement function $\sigma(u^{\mathrm{UB}}, u^{\mathrm{LB}})$ can be 1-norm, i.e., $\mathbf{1}^T(u^{\mathrm{UB}} - u^{\mathrm{LB}})$, where $\mathbf{1}$ is an all-ones vector, or the radius of the Chebyshev ball of the variation range $[u^{\mathrm{LB}}, u^{\mathrm{UB}}]$. Equation (2) describes how system reacts to each uncertainty realization u via the corrective actions $a(u)$. This constraint must hold for any uncertainty realized in the range $[u^{\mathrm{LB}}, u^{\mathrm{UB}}]$. In general, (2) can include any system operational constrains, depending on the applications. Equation (3) indicates that the cost of the corrective actions must not exceed the cost threshold $\bar{C}$ for any realization of uncertainty. The choice of cost threshold depends on decision makers' conservatism level. Compared to a risk-taker, a risk-averse decision maker would be willing to pay more in order to keep system remain reliable with respect to large disturbance, so his cost threshold would be higher. The optimal solution $(u^{*,\mathrm{LB}}, u^{*,\mathrm{UB}})$ of problem (1)–(3) corresponds to lower and upper bounds of the largest range of uncertainty that the system can sustain within the response time window T, and the cost threshold $\bar{C}$.

We define a flexibility metric by comparing the largest variation range with the target range. In an abstract form, the flexibility metric, denoted by F_T, is a function of the tuple $(u^{*,\mathrm{LB}}, u^{*,\mathrm{UB}}, \bar{u}^{\mathrm{LB}}, \bar{u}^{\mathrm{UB}})$. For example, F_T can be defined as follows to indicate whether the system is flexible or not:

$$F_T = \begin{cases} 1, & \text{if } [u^{*,\mathrm{LB}}, u^{*,\mathrm{UB}}] \supseteq [\bar{u}^{\mathrm{LB}}, \bar{u}^{\mathrm{UB}}] \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

When $F_T = 0$, it implies that the system cannot meet the target variation range; when $F_T = 1$, it suggests that the largest variation range is greater than the target one. Another example is to define F_T as the following two indices to show how much upward and downward flexibility the system has:

$$F_T^{\mathrm{UB}} = u^{*,\mathrm{UB}} - \bar{u}^{\mathrm{UB}}$$
$$F_T^{\mathrm{LB}} = \bar{u}^{\mathrm{LB}} - u^{*,\mathrm{UB}}.$$

The larger F_T^{UB} (or F_T^{LB}) is, the more upward (or downward) flexibility the system has. When F_T^{UB} (or F_T^{LB}) is negative, then it implies that the target range cannot be met.

It is worth mentioning that the largest variation range of uncertainty concept is also investigated in [27] from the perspective of process design in chemical plants. In [27], Bansal *et al.* impose some strong assumptions on the variation range and use parametric programming technique to obtain the largest range. Compared with [27], our model (1)–(3) is more general, and we use a robust optimization technique, which will be presented in the next section, to solve the model.

Instead of the hypercube $[u^{\mathrm{LB}}, u^{\mathrm{UB}}]$, we can model the target and largest variation range of uncertainty as an ellipsoid or a polyhedron to better capture the underlying relationship among various sources of uncertainty. Additionally, different choices of the objective function $\sigma(u^{\mathrm{UB}}, u^{\mathrm{LB}})$ in Problem (1)–(3) can reflect different conservatism level of decision makers as well as specific needs of implementations. However, a tradeoff has to be made between computational tractability and model complexity.

Although the choices of the uncertainty set and objective function of problem (1)–(3) are of importance to a meaningful flexibility metric, the purpose of the proposed flexibility metric is to demonstrate one of the possible ways to measure flexibility using the four-element framework. The investigation of various forms of the uncertainty set and objective function is out of the scope of this paper and is an interesting future research direction.

V. Solution Methodology

Here, we describe the solution methodology for problem (1)–(3). It is important to emphasize that problem (1)–(3) is not a standard two-stage robust optimization problem, which can be stylized as follows:

$$\min_{x,a(\bullet)} \left(c_1^T x + \max_{u \in [u^{\mathrm{LB}}, u^{\mathrm{UB}}]} c_2^T a(u) \right)$$
$$\text{s.t.} \quad Mx + Aa(u) + Bu \le b \quad \forall u \in [u^{\mathrm{LB}}, u^{\mathrm{UB}}]. \tag{5}$$

In problem (5), the uncertainty set $[u^{\mathrm{LB}}, u^{\mathrm{UB}}]$ represents the variation range of the uncertain parameter u, and it is pre-selected. The goal of the standard two-stage robust optimization problem is to optimize the system by choosing a first-stage decision variable x so as to accommodate the worst-case realization of u in the uncertainty set $[u^{\mathrm{LB}}, u^{\mathrm{UB}}]$ at the second stage. The robust unit commitment problem in [28] and [29] and robust $\mathrm{N}-\mathrm{k}$ contingency analysis model [30], [31] are examples of standard two-stage robust optimization problems. The Benders type decomposition methods, such as the ones proposed in [28] and [29] can be used to solve problem (5).

On the other hand, problem (1)–(3) is to find the largest uncertainty set that system can accommodate given the system's capability. The upper and lower bounds of the uncertainty set $[u^{\mathrm{LB}}, u^{\mathrm{UB}}]$ are decision variables, instead of pre-determined parameters as in the standard robust optimization problem (5). Therefore, the traditional solution methodologies for the standard robust problem cannot be applied to solve problem (1)–(3) directly.

To solve problem (1)–(3), we adopt the solution methodology proposed in [25]. The idea of the methodology in [25] is to reformulate problem (1)–(3) by using variable substitution, and convert it to an equivalent standard two-stage robust optimization problem. In particular, for any u in the range $[u^{\mathrm{LB}}, u^{\mathrm{UB}}]$, u can be expressed in the following way by introducing a new n-dimensional continuous variable z:

$$u = z \cdot u^{\mathrm{LB}} + (1-z) \cdot u^{\mathrm{UB}} \tag{6}$$

where the operation "$\cdot$" means the component-wise multiplication. Each component of z is between 0 and 1. Substituting (6) into problem (1)–(3), we obtain

$$\max_{u^{\mathrm{LB}}, u^{\mathrm{UB}}, a(\cdot)} \sigma(u^{\mathrm{UB}}, u^{\mathrm{LB}}) \tag{7}$$
$$\text{s.t.} \quad Aa(z) + B(z \cdot u^{\mathrm{LB}} + (1-z) \cdot u^{\mathrm{UB}}) \quad \forall z \in [\mathbf{0}, \mathbf{1}]$$
$$\le b, \tag{8}$$
$$c^T a(z) \le \bar{C}, \quad \forall z \in [\mathbf{0}, \mathbf{1}]. \tag{9}$$

Problem (7)–(9) is a standard two-stage robust optimization problem. The first-stage decision variables are the upper and lower bounds (u^{UB} and u^{LB}) of the variation range while the second-stage decision variable is $a(z)$. Because of (6), the uncertainty in problem (7)–(9) is z, which is characterized by a given uncertainty set $[\mathbf{0}, \mathbf{1}]$, where $\mathbf{0}$ and $\mathbf{1}$ are a zero and all-ones vectors, respectively, with the same dimension as z. As a result, we transform the non-standard robust optimization problem (1)–(3) into a standard robust problem (7)–(9). If the objective function $\sigma(u^{\mathrm{UB}}, u^{\mathrm{LB}})$ is a linear or concave function, problem (7)–(9) can be solved by using the approaches proposed in [28] and [29] for continuous corrective action $a(z)$ such as dispatch and AGC control or, alternatively, the algorithm proposed in [32] for discrete corrective action, such as unit commitment and transmission line switching. Although the solution methodology developed in [25] is used to solve problem (1)–(3), there is a significant difference between [25] and this paper. The focus of [25] is to design a dispatch framework for variable resources through a Do-Not-Exceed limit, which is the largest variable resources' output the system can accommodate, whereas the main goal of this paper is to establish a theoretical framework to analyze system flexibility.

The proposed solution methodology has the following limitation. If the uncertain parameter u is integer, we cannot convert problem (1)–(3) to problem (7)–(9) via (6). A different approach needs to be developed to solve problem (1)–(3) with integer uncertain parameter, which is a very challenging problem in itself, and needs to be addressed in a separate paper.

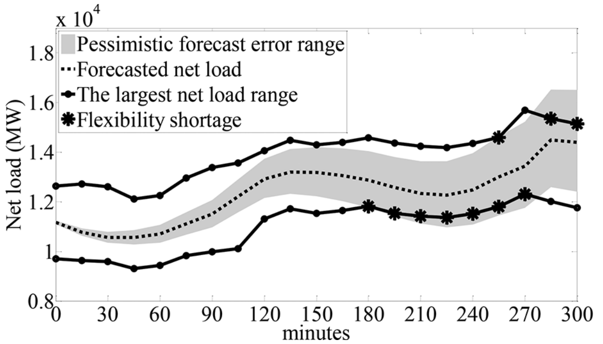

Fig. 2. Flexibility under the pessimistic forecast error range.

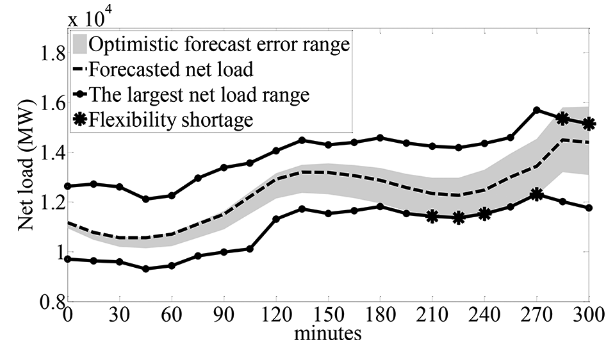

Fig. 3. Flexibility under the optimistic forecast error range.

VI. Real-Time Operation Application

Here, we apply the flexibility metric proposed in Section IV to the real-time operation in the presence of load and wind output uncertainty. In the numerical example, we demonstrate how the proposed flexibility metric is used as a situational awareness tool for the operator to identify potential flexibility shortage during the operation time horizon. We also show how the two essential elements *uncertainty* and *cost* affect the flexibility metrics, as well as propose a proactive control action to avoid the flexibility shortage.

In the example, we assume that at the initial time 0, operators look 300 min of 5 h ahead and want to know whether the system will have the capability to accommodate the uncertain future load and wind output fluctuation given the current state of system and the ramping capability of the online units. For simplicity, the available corrective action is restricted to be economic dispatch.

At initial time 0, the forecast of the expected future net load, which equals the aggregated load less the aggregated wind output, is available to operators. By using the stochastic properties of load and wind forecast, for example choosing a certain confidence interval level, operators can obtain a forecast error range around the net load forecast. The forecast error range is the target variation range. The magnitude of the forecast error range reflects the operators' risk level. If an operator is risk-averse or pessimistic about the forecast accuracy, then the forecast error range will be larger than if the operator is optimistic. The grey shaded areas in Figs. 2 and 3 illustrate the pessimistic and optimistic forecast error ranges, respectively. For both cases, the forecast error range broadens as we assume that the forecast accuracy decreases with time horizon.

Problem (1)–(3) can be casted into the following specific form, which represents finding the largest net load range that the system can accommodate for a given response time T, and a response cost threshold $\bar{C}_T$. We choose the radius of the Chebyshev ball $r(d_{n,T}^{\mathrm{UB}}, d_{n,T}^{\mathrm{LB}})$ to measure the size of variation range in the objective

$$\max_{d^{\mathrm{UB}},d^{\mathrm{LB}},p(\cdot)} \sum_{n=\mathrm{RAND}} r(d_{n,T}^{\mathrm{UB}}, d_{n,T}^{\mathrm{LB}}) \quad (10)$$

$$\text{s.t.} \quad \sum_{j=\mathrm{CCU}} p_{j,T}(\mathbf{d}) - \sum_{n=\mathrm{RAND}} d_{n,T} = 0 \qquad \forall \mathbf{d} \in [\mathbf{d}^{\mathrm{LB}}, \mathbf{d}^{\mathrm{UB}}] \quad (11)$$

$$\sum_n SF_{n,l} \times (p_{j(n),T}(\mathbf{d}) - d_{n,T}) \le F_l^{\max} \qquad \forall l = \mathrm{TL},\ \forall \mathbf{d} \in [\mathbf{d}^{\mathrm{LB}}, \mathbf{d}^{\mathrm{UB}}] \quad (12)$$

$$p_{j,0} - \Delta_j^{\mathrm{dn}} \times T \le p_{j,T}(\mathbf{d}) \le p_{j,0} + \Delta_j^{\mathrm{up}} \times T \qquad \forall j = \mathrm{CCU}, \quad \forall \mathbf{d} \in [\mathbf{d}^{\mathrm{LB}}, \mathbf{d}^{\mathrm{UB}}] \quad (13)$$

$$p_{j,T}^{\min} \le p_{j,T}(\mathbf{d}) \le p_{j,T}^{\max} \qquad \forall j = \mathrm{CCU}, \forall \mathbf{d} \in [\mathbf{d}^{\mathrm{LB}}, \mathbf{d}^{\mathrm{UB}}] \quad (14)$$

$$\sum_{j=\mathrm{CCU}} c_j \times p_{j,T}(\mathbf{d}) \le \bar{C}_T \qquad \forall \mathbf{d} \in [\mathbf{d}^{\mathrm{LB}}, \mathbf{d}^{\mathrm{UB}}] \quad (15)$$

where $\mathbf{d}$ is the vector of all of the $d_{k,T}$'s. The objective function (10) is to maximize the total net load variation range. Constraints (11) and (12) are the balance and transmission constraints, which need to be satisfied for any uncertain net load d_T realized in the interval $[d^{\mathrm{LB}}, d^{\mathrm{UB}}]$. We assume that any units that are not subject to uncertainty are the corrective control units (CCUs), which operators dispatch to provide corrective control action to the variation of the net load. Therefore, their output $p_{j,T}(\mathbf{d})$ is dependent on the uncertain net load. Constraint (13) indicates the upward and downward ramping capability of CCUs within the response time window. Constraint (14) is the resource capacity constraint. The last constraint imposes a limit on the total dispatch cost.

We use the real-world power system operated by ISO New England. There are about 150 CCUs and six wind generators. The total wind capacity is artificially scaled up to 2479 MW in this example. We choose 15 minutes as time resolution, and therefore, solve problem (10)–(15) for T equal to 15, 30, ..., 300 min, respectively. The results of problem (10)–(15) are used to measure the system flexibility, instead of changing the course of dispatch actions over the time horizon. However, based on the results of problem (10)–(15), some additional proactive actions can be taken to change the dispatch path, which will be discussed in Section VI-C.

A. Impact of Uncertainty

Here, we study the impact of target uncertainty range on the flexibility metric. The cost limit is uniformly set to be sufficiently large ($\bar{C}_T = \$10^7$) for all T's. The resulting largest net load variation range (the area between the solid curves) over the 5-h time horizon is compared to the pessimistic and optimistic forecast error ranges in Figs. 2 and 3, respectively. A flexibility shortage occurs when the upper (lower) bound of the variation range is below (above) the upper (lower) bound of the forecast error range. The asterisks flag the time when there is a flexibility shortage. The comparison shows that the frequency of flexibility shortage is subject to the risk tolerance of operators because the target uncertainty range or the forecast error range is subjectively set by operators.

When operators are pessimistic about the forecast, the asterisks at the lower solid curve in Fig. 2 indicate that the system does not have enough downward ramping capability from minute 180 to 270. The reasons for the downward ramping shortage are two. First, the expected net load is trending down from minute 180 to minute 225. Second, the forecast error range is significantly broadened from minute 180 in the pessimistic case as shown in Fig. 2. The asterisks on the upper solid curve in Fig. 2 suggest that the system experiences upward ramping capability shortage at the last few time periods. This is because of the increase in net load as well as the considerable forecast error at those time periods as illustrated in Fig. 2. In addition, if the possible net load realizations are assumed to vary anywhere within the pessimistic forecast error range in Fig. 2, then the asterisks can indicate the net load level that will trigger the flexibility shortage events.

On the other hand, when operators' risk tolerance is high, i.e., the optimistic case, they will perceive that the system is less likely to have ramping shortages although the actual ramping capability does not change. This is because the optimistic forecast error range is much narrower in Fig. 3 compared with the pessimistic case in Fig. 2. As a result, system has sufficient upward and downward flexibility most time in the 5-h look-ahead time horizon, except a few time periods between minute 210 and 300, as shown in Fig. 3. In other words, when the perceived possible net load realization is very close to the forecast value, the system is deemed to be more flexible in Fig. 3 than Fig. 2. This implies that the degree of flexibility depends on decision makers' risk preference level, which can be quantified by the target variation range.

B. Impact of Cost

Here, we investigate how the cost limit of corrective actions affects the flexibility metric. To this end, we reduce the cost limit from $\$10^7$ used in Section VI-A to only additional 1% of the dispatch cost of meeting the forecasted net load. We show that the smaller the cost limit, the narrower the largest variation range. Moreover, the upper bound of the variation range is more affected by the cost limit than the lower bound.

The largest net load variation range with the low cost limit is plotted as the curves with triangle markers while the high cost limit counterpart is plotted as the solid curves in Fig. 4. It can be seen that the low-cost limit significantly reduces the upper bound of the largest net load variation range. This implies that, due to the tight cost restriction on corrective actions, the system has less upward ramping capability to accommodate the net load varying above the forecasted value, as compared to its capability under the high cost limit. As a result, there will be more flexibility shortage events under the tight cost limit. On the other hand, the cost limit has little impact on the lower bound of the largest variation range. This suggests that the system's downward ramping capability is less affected by the cost than

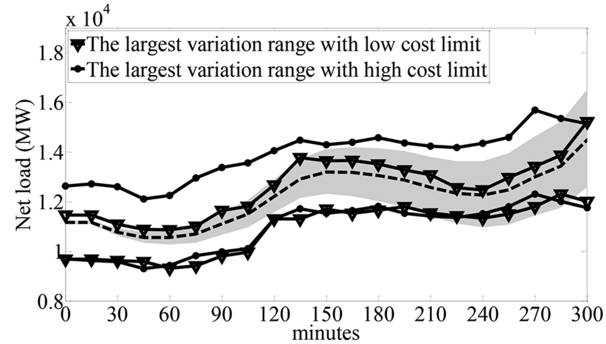

Fig. 4. Impact of cost limit on the largest net load variation range.

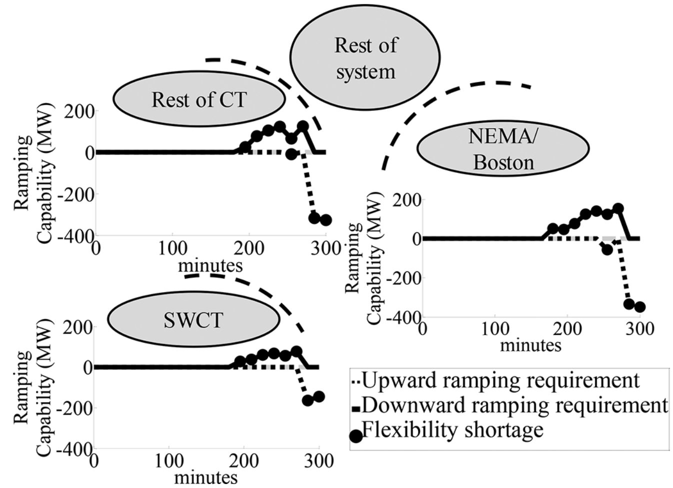

Fig. 5. Zonal ramping requirements.

the upward ramping capability. The reason is that it takes energy for a resource to generate power to ramp up whereas it costs little for a resource to reduce output to ramp down.

C. Proactive Control

By visualizing the flexibility metric as demonstrated in Figs. 2 and 3, operators can easily spot the flexibility shortage events in advance. Combined with the deterministic look ahead multi-interval unit commitment and dispatch tools [33], [34], operators can take proactive actions, such as recommitting units, repositioning units or adjusting ramping or reserve requirements if they are in place to resolve the shortfall before it happens. Here, we focus on the determination of zonal ramping requirement using the concept of the largest net load variation range.

More specifically, the solutions to Problem (10)–(15) provide the largest net load variation range at each node. Since transmission constraints are taken into account in Problem (10)–(15), we can obtain the zonal largest net load variation ranges by aggregating the nodal ranges associated with each zone. By comparing the zonal variation range with the forecast error range, we can determine how much upward and downward ramping capability is needed for each zone, which can be translated into the zonal ramping requirements for a flexibility shortage event.

There are three load pockets areas, namely northeast Massachusetts (NEMA)/Boston, southwest Connecticut (SWCT),

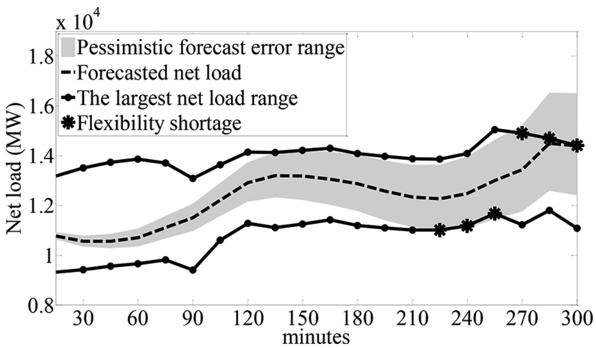

Fig. 6. Flexibility under the pessimistic forecast error range with zonal ramping requirements.

and the rest of Connecticut (CT), in ISO New England control area. In this numerical experiment, we concentrate on the zonal ramping requirements in these three areas in Fig. 5. The plots are based on results from the high cost limit and pessimistic forecast case. The dotted curve corresponds to the upward ramping requirements while the solid one is for the downward ramping requirement. The black dots highlight the flexibility shortage events. As shown in Fig. 5, the ramping requirements are zero when flexibility is not deficient. From minute 180 to 300 when flexibility shortage occurs, Fig. 5 indicates how much upward and downward ramping capability is needed for each zone. By using this information, system operators can set zonal ramping requirements before a predicted flexibility shortage event. In this way, the system can prepare sufficient ramping capability at where it is needed to avoid a possible flexibility shortage in the future.

To demonstrate the value of the zonal ramping requirement approach, we carry out the following experiment to compare the system flexibility with and without zonal ramping requirements. The zonal ramping requirements as shown in Fig. 5 are imposed in the economic dispatch at minute 15. This is to simulate the situation that operators observe the predicted flexibility shortages at the beginning time 0; then they take proactive action by dispatching the system at the next time interval, minute 15, taking into account the identified ramping requirements. Using the modified dispatch solution at minute 15, we recalculate the largest net load variation ranges from minutes 15 to 300. The resulting flexibility curves are plotted in Fig. 6, which are analogous to Fig. 2. It can be observed that the largest net load range in Fig. 6 deviates from the one in Fig. 2 because system is repositioned under the zonal ramping requirement approach. As a result, this approach eliminates the downward flexibility shortage events occurring at minutes 180, 195, 210 and 270 as well as the upward shortage event at minute 255. Because the resources' ramping capability is limited by their ramp rate and capacities, the ramping requirement approach does not significantly broaden the largest net load variation range. Instead, it strategically repositions the system at an early stage so that some flexibility issues can be avoided down the road. Although the zonal ramping requirement approach may not completely resolve all the flexibility shortage events, comparison between Figs. 2 and 6 indicates that such an approach can effectively deter early flexibility deficit in a chain of shortage events. The associated benefit is that it buys additional time for operators to adopt other means to solve flexibility issues in the near future.

Additionally, the computation time of solving problem (10)–(15) is 3.57 s on average in the example. The efficient computation can allow operators to use up-to-date forecast and to refresh the system flexibility information on a frequent basis.

VII. Conclusion

Although flexibility has been extensively discussed and investigated in the industry and academia in recent years, the current study of flexibility concept is still in its infancy. In particular, there is a lack of unified framework for measuring multiple aspects of flexibility in power systems. This paper is an effort to provide such a framework, based on the common fundamental elements that define the nature of flexibility in different aspects of power systems. The fundamental elements of flexibility are time, action, uncertainty, and cost. Based on the four-element framework, we propose a flexibility metric that reflects the excessive availability of the system relative to a target range. Compared to the existing work, the proposed metrics explicitly takes into account the impacts of transmission network and operation constraints, which play an important role in affecting the system flexibility. A robust optimization technique is used to calculate the metrics, so it reduces the burden of gathering accurate stochastic information. We apply the proposed framework to the real-time operation using the ISO New England power system in the presence of wind output and load uncertainty. The visualized flexibility metric serves as a situational awareness tool for operators to identify the potential flexibility shortage events in the operation time horizon. In addition, it also provides useful information on determining zonal ramping requirements to procure ramping capability at where it is needed.

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